Probabilistic properties of model-based diagnostic reasoning in Bayesian networks

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Abstract

So far, much research has been carried out into general Bayesian networks and limited attention has been given to special types of Bayesian networks for specific applications. An example of such an application is model-based diagnosis, i.e. the diagnosis of malfunctioning of devices or systems, based on an explicit model of the structure and behaviour of these devices and systems. Basically, two types of model-based diagnosis are being distinguished: (*i*) consistency-based diagnosis, and (*ii*) abductive diagnosis.

In this paper, we investigate the relationship between consistency-based and abductive reasoning in Bayesian networks. It appears that abductive diagnoses can be determined using results from consistency-based diagnosis. Furthermore, we derive special properties of Bayesian networks for model-based diagnostic reasoning, and using these properties we derive a theoretically and a computationally simplified form for both consistency-based and abductive reasoning in probabilistic representations.

1 Introduction

In the early 1980s, Bayesian networks were introduced as a special type of probabilistic graphical models representing uncertain knowledge in a both qualitative and quantitative way [5]. Much research has been carried out in the field of Bayesian networks since then, mostly focusing on general properties of these network models. More recently, researchers have started focusing on special types of Bayesian networks that enable us to gain insight into the special properties of Bayesian networks for specific applications.

In the last two decades, model-based diagnosis has become an increasingly important area of research, mainly because the complexity of systems has risen considerably and trouble shooting of faults in such systems has therefore seen a similar rise in complexity. Basically, two types of model-based diagnosis are being distinguished in literature: (*i*) consistency-based diagnosis [2, 8], and (*ii*) abductive diagnosis [6]. Consistency-based diagnosis is based on models of the correct behaviour of a system under investigation, whereas abductive diagnosis focuses on the use of fault models of systems.

Several researchers have attempted to clarify the relationship between abductive and consistency-based diagnosis. In their seminal paper, Console and Torasso have shown that these two notions of diagnosis allow one to span an entire spectrum of diagnostic notions, where pure consistency-based and abductive diagnosis act as extremes [1]. Although pure abductive and consistency-based diagnosis are different, their formalisation has also many similarities, which explains why it is possible to develop such a spectrum.

Flesch et al. have developed a method to add uncertainty reasoning to consistency-based diagnosis exploiting Bayesian networks [4], where logical notions of inconsistency and consistency are taken to correspond to a probability equal and nonequal to 0. The ideas underlying the logical spectrum proposed by Console and Torasso makes one wonder whether it is possible to do something similar in a probabilistic setting.

More in particular, we are interested in the relationship between abductive diagnosis and consistency-based diagnosis in terms of their underlying theory.

There have been several proposals to utilise Bayesian networks in the context of abductive diagnosis. For example, Pearl [5] has proposed using a special type of Bayesian network for that purpose. Shimony and colleagues have described abductive diagnosis using Bayesian networks in terms a maximum a-posteriori assignment, which is known to be NP hard [9]. As far as we know, no one before has studied the relationship between probabilistic abductive diagnosis and consistency-based diagnosis in a way resembling the logical spectrum of model-based diagnosis. The establishment ot this relationship is the topic of this paper.

The paper is organised as follows. In Section 2, the necessary basic concepts are defined, which are followed by the introduction of the concepts of Bayesian diagnostic systems and Bayesian diagnostic problems. Subsequently, in Section 3 the relation between consistency-based and abductive diagnostic reasoning in Bayesian diagnostic problems are explored. Finally, in Section 5, the paper is rounded off with conclusions and future work.

2 Preliminaries

2.1 Model-based diagnosis

Two types of model-based diagnosis are distinguished in literature: (*i*) consistency-based diagnosis [2, 8], and (*ii*) abductive diagnosis [6]. In the theory of consistency-based diagnosis [8], the structure and behaviour of a system is represented by a *logical diagnostic system* $\mathcal{S}_L = (\mathrm{SD}, \mathrm{COMPS})$, where

- SD denotes the *system description*, which is a finite set of logical formulae, specifying structure and behaviour;
- COMPS is a finite set of constants, corresponding to the *components* of the system; these components can be faulty.

The system description consists of *behaviour descriptions*, and *connections*. A behavioural description is a formula specifying *normal* and *abnormal* (faulty) functionalities of the components. These normal and abnormal functionalities are indicated by *abnormality literals*. A connection is a formula of the form $i_c = o_{c'}$, where i_c and $o_{c'}$ denote the input and output of components c and c'.

A *logical diagnostic problem* is defined as a pair $\mathcal{P}_L = (\mathcal{S}_L, \mathrm{OBS})$, where $\mathcal{S}_L$ is a logical diagnostic system and OBS is a finite set of logical formulae, representing *observations*.

Adopting the definition from [3], a diagnosis in the theory of consistency-based diagnosis is defined as follows. Let Δ_C be the assignment of abnormal behaviour to the set of of components $C \subseteq COMPS$ and normal behaviour to the remaining components of the system. Then, Δ_C is a *consistency-based diagnosis* of the logical diagnostic problem $\mathcal{P}_L$ iff the observations are consistent with both the system description and the diagnosis:

$$\mathrm{SD} \cup \Delta_C \cup \mathrm{OBS} \nvDash \bot.$$

Here, $\nvDash$ stands for the negation of the logical entailment relation, and $\bot$ represents a contradiction.

Abductive diagnosis focuses on the use of fault models of systems, where the observations have to be *implied* by the diagnosis. The behavioural assumption Δ_C is called an *abductive diagnosis* if the system description and the behavioural assumption imply the set of observations:

$$\mathrm{SD} \cup \Delta_C \vDash \mathrm{OBS}.$$

EXAMPLE 1 Consider Figure 1 (a), which depicts an electronic circuit with one AND gate and two OR gates. Now, the output of the system differs from the one expected according to the simulation model, thus it gives rise to an inconsistency. One of both consistency-based and abductive diagnoses is to assume that the AND gate is functioning abnormally. □

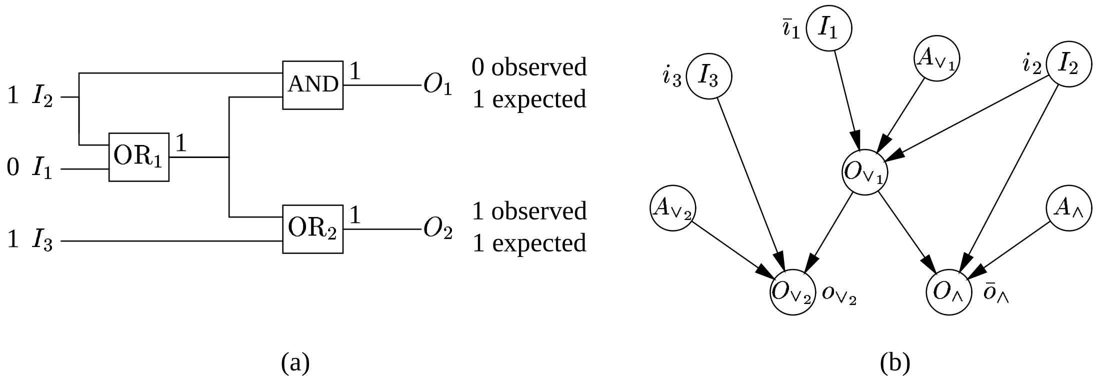

Figure 1: (a) Example of a circuit. (b) A Bayesian diagnostic system corresponding to circuit (a).

2.2 Bayesian networks and Bayesian diagnostic problems

Let $P(X)$ be a joint probability distributions of the set of discrete binary random variables X, where singleton random variable $\{Y\} \in X$ is abbreviated to Y and it takes the values 'true' or 'false', written as y and $\bar{y}$, respectively. However, if we refer to arbitrary values of a set of variables X, not a single variable only, this will be denoted by x as well. Let $U, W, Z \subseteq X$ be disjoint sets of random variables, then U is said to be *conditionally independent* of W *given* Z, if

$$P(U \mid W, Z) = P(U \mid Z) \tag{1}$$

A *Bayesian network* $\mathcal{B}$ is defined as a pair $\mathcal{B} = (G, P)$, where $G = (V, E)$ is an acyclic directed graph, with set of vertices V and set of arcs E, P is the associated joint probability distribution on the set of random variables X, and G represents the relations of X satisfying the condition that each represented independence relation is also valid in the joint probability distribution implying factorisation:

$$P(X) = \prod_{Y \in X} P(Y \mid \pi(Y)) \tag{2}$$

where $\pi(Y)$ denotes the parent set of vertex Y. There exists a 1–1 correspondence between the vertices V and the random variables X, and we will therefore use the same notation for both. Finally, we will frequently make use of marginalising out particular variables W written as $P(u) = \sum_w P(u, w)$.

A *Bayesian diagnostic system* is denoted by $\mathcal{S}_B = (G, P)$, where P is a joint probability distribution of the vertices of G, interpreted as random variables, and G is obtained by mapping on logical diagnostic system $\mathcal{S}_L = (\mathrm{SD}, \mathrm{COMPS})$ as follows: (*i*) component c is represented by its input I_c and output O_c vertices, where inputs are connected by an arc with the outputs pointing to the output, (*ii*) to each component c there belongs an abnormality vertex A_c which has an arc pointing to the output of the corresponding component. As a consequence of the independences that hold for a Bayesian diagnostic system, it is possible to simplify the computation of the joint probability distribution P by the following properties:

Property 1: the joint probability distribution of a set of output variables O can be factorised as follows:

$$P(o) = \sum_{i,a} P(o, i, a) = \sum_{i,a} P(i, a) \prod_{c \in \mathrm{COMPS}} P(O_c \mid \pi(O_c)). \tag{3}$$

Property 2: $P(i, a) = P(i)P(a)$, due to the fact that input variables and abnormality variables are mutually independent of each other.

Recall that logical diagnostic problems are logical diagnostic systems augmented with the set of observations; this has to be also the case for Bayesian diagnostic problems. In logical diagnostic systems, observations are the inputs and outputs of a system. The set of input and output variables that have been observed, are referred to as I_ω and O_ω, respectively. The unobserved input and output variables will be refered to as I_u and O_u respectively. We will use the notation i_ω to denote the values of the observed inputs, and o_ω for the observed output values. The set of observations is then denoted as $\omega = i_\omega \cup o_\omega$.

A Bayesian diagnostic problem, denoted by $\mathcal{P}_B = (\mathcal{S}_B, \omega)$, consists of (*i*) a Bayesian diagnostic system representing the components, including their behaviour and interaction, of logical diagnostic system of concern, and (*ii*) a set of observations ω [5, 4].

In Bayesian diagnostic problems the probability of an output value for a normally functioning components is equal to 0 or 1, whereas the probability of an output value for an abnormally functioning component c becomes independent of its parents with the exception of its abnormality a_c.

3 Model-based diagnostic reasoning in Bayesian diagnostic problems

3.1 The relation between abductive and consistency-based reasoning

In this section, we discuss how abductive and consistency-based approaches can be expressed as Bayesian diagnostic problems, and, subsequently, we establish their relationship.

To facilitate a connection between model-based diagnosis using a logical system and diagnostic reasoning using a Bayesian diagnostic system, let C be the set of components assumed to be abnormally functioning, then the corresponding *abnormality assumption* is described as:

$$\delta_C = \{a_c \mid c \in C\} \cup \{\bar{a}_c \mid c \in \mathrm{COMPS} - C\}$$

Translating consistency-based approach to our probabilistic diagnostic theory, the consistency condition requires that the probability of the occurence of the observations given the diagnosis is non-zero. Consequently, if the probability is equal to zero, the set of abnormality assumptions is assumed to be inconsistent. Formally, in consistency-based reasoning, we are searching for probabilities $P(i_\omega, o_\omega \mid \delta_C) > 0$. Observe that we condition on the set of abnormality assumptions δ_C, since it is given knowledge.

In abductive reasoning, the observations have to be implied by the system descriptions and the abnormality assumptions. This means that we are looking for abnormality assumption δ_C that can be explained by the set of observed inputs and outputs, formally, $P(\delta_C \mid i_\omega, o_\omega)$. Considering the formulae for the abductive and consistency-based approaches to diagnosis using Bayes' rule, we obtain, assuming $\alpha = 1/P(i_\omega, o_\omega)$:

$$P(\delta_C \mid i_\omega, o_\omega) = \frac{P(i_\omega, o_\omega \mid \delta_C)P(\delta_C)}{P(i_\omega, o_\omega)} = \alpha \cdot \; P(i_\omega, o_\omega \mid \delta_C)P(\delta_C)\ . \tag{4}$$

Note that by abductive reasoning to obtain a better result we try to maximalise the probability $P(\delta_C \mid i_\omega, o_\omega)$.

Equation (4) establishes the relationship between the consistency-based and abductive approach to Bayesian diagnostic problems. The signifi cance of this formula is as follows. In order to compute abductive diagnoses, it is required to compute the set of consistency-based diagnoses by

$$P(i_\omega, o_\omega \mid \delta_C) = \sum_{i_u} P(i_u)P(i_\omega, o_\omega \mid i_u, \delta_C),$$

where the basic axioms of probability theory and factorisation lead to:

$$\begin{aligned}
P(i_\omega, o_\omega \mid i_u, \delta_C) &= \frac{P(i_\omega, o_\omega, i_u, \delta_C)}{P(i_u, \delta_C)} = \frac{\sum_{o_u} P(o_\omega, o_u, i_\omega, i_u, \delta_C)}{P(i_u, \delta_C)}, \\
P(o_\omega, o_u, i_\omega, i_u, \delta_C) &= P(i, \delta_C)\prod_c P(O_c \mid \pi(O_c)) = P(i_\omega)P(i_u, \delta_C)\prod_c P(O_c \mid \pi(O_c)).
\end{aligned}$$

Often it is helpful to explicit specify what values *some* of the variables in $\pi(O_c)$ take, written by $\pi(O_c)$: *values*. If the input and output variables of a component have been observed, this will be indicated by $I_{c,\omega}$

and $O_{c,\omega}$, respectively. When unobserved, the notation $I_{c,u}$ and $O_{c,u}$ is employed. So we obtain,

$$P(i_\omega, o_\omega \mid \delta_C) = \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(O_c \mid \pi(O_c) : i_{c,u}). \tag{5}$$

When considering the formula of consistency-based reasoning in Equation (5), many terms can be simplified by exploiting specific relations that hold for Bayesian diagnostic systems, done below.

3.2 Decomposition of the joint probability distribution

Formally, groups or classes of probabilistically equivalent components are viewed as equivalence classes. If two components $c, c' \in \text{COMPS}$ belong to the same equivalence class q, also denoted by $[c]$, then for each $c, c' \in [c]$ it holds that the probabilities of the abnormality behaviour of these two components is the same. The set of all equivalence classes will be denoted by Q in the following. Probabilities $P(o_c \mid a_c)$ will be denoted by p_c or p_q; where p_q stresses the fact that a probability is associated to a class of components.

In the analysis that will follow, we will distinguish between several sets of components:

- The sets of components assumed to function normally and abnormally will be denoted by $C^{\bar{a}}$ and C^a.
- The sets $C^{\bar{a}}$ and C^a are partitioned into sets of components, for observed and unobserved outputs, indicated by the sets $C^{\bar{a}}_\omega$, $C^{\bar{a}}_u$, C^a_ω and C^a_u, respectively. Thus, $C^{\bar{a}} = C^{\bar{a}}_\omega \cup C^{\bar{a}}_u$ and $C^a = C^a_\omega \cup C^a_u$.

In addition, we will make a distinction between components c for which o_c has been observed, and components c for which $\bar{o}_c$ has been observed. These sets will be denoted by C^o_ω and $C^{\bar{o}}_\omega$, respectively. It holds that C^o_ω and $C^{\bar{o}}_\omega$ constitute a partition of C_ω. The notations can also be combined, e.g., as $C^{a,o}_\omega$ and $C^{a,\bar{o}}_\omega$. Furthermore, we will sometimes use a similar notation for sets of output variables, e.g., $O^{\bar{a}}_u = \{O_c \mid c \in C^{\bar{a}}_u\}$ and $O^{\bar{a}}_\omega = \{O_c \mid c \in C^{\bar{a}}_\omega\}$. In the following three consecutive sections, we introduce properties of the Bayesian diagnostic problems that allow us to simplify probabilistic terms for the computation of the consistency-based probability.

3.2.1 Decomposition of the set of output variables

The following lemma shows that it is useful to explicitly distinguish between various types of components using the component sets defined above.

Lemma 1 *The joint probability distribution of the outputs of the set of assumed* normally *functioning components $C^{\bar{a}}$, can be decomposed into two products as follows:*

$$\prod_{c \in C^{\bar{a}}} P(O_c \mid \pi(O_c) : \bar{a}_c) = \prod_{c \in C^{\bar{a}}_u} P(O_c \mid \pi(O_c) : \bar{a}_c) \prod_{c \in C^{\bar{a}}_\omega} P(O_c \mid \pi(O_c) : \bar{a}_c).$$

This lemma is also similar for the set of abnormally functioning components C^a.

3.2.2 Observed abnormally assumed output variables

In this section, we establish that the outputs of the set of observed abnormally components depend only on their probability of abnormal behaviour and not on other components in their parent set.

Lemma 2 *Let Q be the set of equivalence classes of components with identical faulty behaviour probabilities. The joint probability of observed outputs of the abnormally assumed components is written as:*

$$\prod_{c \in C^a_\omega} P(O_c \mid \pi(O_c) : a_c) = \prod_{q \in Q} p_q^{n_q} (1 - p_q)^{m_q},$$

where $n_q = |\{c \mid c \in q \cap C^{a,o}_\omega\}|$, i.e., the number of abnormal components with positive output and probability p_q; similarly, for negative outputs we have $m_q = |\{c \mid c \in q \cap C^{a,\bar{o}}_\omega\}|$.

3.2.3 Normally assumed output components as Boolean functions

Recall that the probability of an output of a normally functioning component was assumed to be either 0 or 1 implying that these probabilities act as Boolean functions. Straightforward, if these probabilities individually can be seen as Boolean functions, then the product over these probabilities is also a Boolean function. Let Boolean function φ be defined as the product of probabilities of output values of a given subset of normally functioning components; formally, let $C^{\bar{a}'} \subseteq C^{\bar{a}}$, then

$$\varphi(C_u^{\bar{a}'}) = \prod_{c \in C_u^{\bar{a}'}} P(O_c \mid \pi(O_c) : \bar{a}_c).$$

Since φ is a Boolean function it is either equal to 0 or to 1. Furthermore, it is a product of Boolean functions all related to single components. Note that for each single Boolean component there is only one output value for which the probability is equal to 1 (for the same values of the parent set). This indicates that for the product of these single Boolean functions there is also only one output value combination for which the product of probabilities is equal to 1 and, therefore, for which φ is equal to 1.

Lemma 3 *There exists only* one *value instantiation $o_u^{\bar{a}}$ of the set of variables $O_u^{\bar{a}} = \{O_c \mid c \in C_u^{\bar{a}}\}$ for which it holds that $\varphi(C_u^{\bar{a}}) = 1$; similarly, there exists one value $o_\omega^{\bar{a}}$ of the set of variables $O_\omega^{\bar{a}} = \{O_c \mid c \in C_\omega^{\bar{a}}\}$ for which it holds that $\varphi(C_\omega^{\bar{a}}) = 1$.*

3.2.4 Relation between abnormally and normally functioning outputs

In this section, the summation over O_u of the consistency-based approach in Equation (5) will be simplified, for which we devide the set of unobserved outputs O_u into two subsets: outputs of normally and abnormally functioning components. In the previous sections we showed that probabilities of normally functioning components are Boolean functions, and those for abnormally functioning components can be written as a product over the abnormality probabilities. Then, in the summation over O_u, a form of the product of abnormality probabilities can occure several times, and each time multiplied by a Boolean function.

Lemma 4 *Let the Boolean function φ be as defined above, then,*

$$\sum_{o_u} \prod_{c} P(O_c \mid \pi(O_c) : i_{c,u}) = \sum_{o_u} \varphi(C^{\bar{a}}) \prod_{c \in C_u^a} P(O_c \mid a_c) = \sum_{o_u^a} b(o_u^a) \prod_{q \in Q} p_q^{n_q} (1 - p_q)^{m_q},$$

where $b(o_u^a)$ is the number of Boolean functions, which counts the values of $\varphi(C^{\bar{a}})$ for the related product form in q, n_q is the number of abnormal components with positive output and probability p_q; similarly, for negative outputs we have m_q.

Based on probability theory, if we take the sum over the probabilities over output values that do not occur in the domain of the Boolean functions φ, these probabilities can be summed out.

Lemma 5 *Let $C_u^a = C_u^{a\prime} \cup C_u^{a\prime\prime}$ be a partition of the set of components C_u^a, as defined above, such that $C_u^{a\prime} = \{c \mid c \in C_u^a, \forall c' \in C^{\bar{a}} : O_c \notin \pi(O_{c'})\}$, i.e., abnormally assumed components with unobserved outputs that do not act as input to any normally assumed component, then $C_u^{a\prime\prime} = C_u^a - C_u^{a\prime}$. Then,*

$$\sum_{o_u} \prod_{c \in C^{\bar{a}}} P(O_c \mid \pi(O_c) : \bar{a}_c) \prod_{c \in C_u^a} P(O_c \mid a_c) = \sum_{o_u - o_u^{a\prime}} \prod_{c \in C^{\bar{a}}} P(O_c \mid \pi(O_c) : \bar{a}_c) \prod_{c \in C_u^{a\prime\prime}} P(O_c \mid a_c).$$

3.3 Simplified forms for model-based reasoning

Based on the results above, the following theorem and corollary yield a simplified computational form for consistency-based and abductive diagnoses.

Theorem 1 *Let $\mathcal{P}_B = (\mathcal{S}_B, \omega)$ be a Bayesian diagnostic problem. Then,*

$$P(i_\omega, o_\omega \mid \delta_C) = \sum_{i_u} P(i_u) \sum_{o_u^{a''}} b(o_u^{a''}) \prod_{q \in Q} p_q^{n_q} (1 - p_q)^{m_q} \ .$$

Proof: Combination of lemmas 4 and 5 yields the summation over $O_u^{a''}$ and its probabilistic terms. □

Corollary 1 *Let $\mathcal{P}_B = (\mathcal{S}_B, \omega)$ be a Bayesian diagnostic problem, and let $\alpha = 1/P(i_\omega, o_\omega)$. Then,*

$$P(\delta_C \mid i_\omega, o_\omega) = \alpha \cdot \prod_{c \in C} P(\delta_c) \cdot \sum_{i_u} P(i_u) \sum_{o_u^{a''}} b(o_u^{a''}) \prod_{q \in Q} p_q^{n_q} (1 - p_q)^{m_q} \ .$$

Proof: It follows directly from Equation (4) and from Theorem 1. □

Based on Theorem 1 and Corollary 1, firstly, abductive diagnoses in Bayesian diagnostic problems can be determined using consistency-based diagnosis. Secondly, diagnostic reasoning does not have to take into account the entire set of outputs of components leading to a simplified computational formula, thus, we do not have to know all the abnormal behaviours still we can reason in an *exact* way.

4 Logical and probabilistic model-based diagnosis related

In this section, we establish the connection between logical and probabilistic model-based diagnosis based on the above-presented results.

To start, we would like prove the following statement:

$$P(\omega \mid \delta_C) > 0 \Longleftrightarrow SD \cup OBS \cup \Delta_C \nvDash \bot \ .$$

Recall that consistency-based diagnosis in Bayesian diagnostic problems is expressed by the term $P(\omega \mid \delta_C) > 0$. Applying the Bayes rule, consistency-based diagnosis requires the following two requirements: (*i*) $P(\delta_C) > 0$ and (*ii*) $P(\omega, \delta_C) > 0$. For the first item (*i*) it holds that

$$\begin{aligned} P(\delta_C) > 0 \quad &\Longleftrightarrow \quad P(a_1, a_2, \ldots, \bar{a}_{|COMPS|}) > 0 \\ &\Longleftrightarrow \quad P(a_1)P(a_2) \cdots P(a_{|C|}) \neq 0 \ \wedge \ P(\bar{a}_{|C|+1}) \cdots P(\bar{a}_{|COMPS|}) > 0 \ , \end{aligned} \tag{6}$$

where the abnormality assumptions in δ_C are sorted in the way that the first part consists of the abnormal behavioral assumptions, whereas the second part includes the normal behavioural assumptions. Note that in the logical reasoning, int the probabilistic terms $P(a_c)$ and $P(\bar{a}_c)$ the probability distribution P is related to the system description, where the abnormal and normal functionality of the systems is defined, whereas the abnormal and normal behaviour is related to Δ_c. The requirement $P(\Delta_C) > 0$ corresponds in logic with the fact that by definition $\Delta_C \nvDash \bot$, since Δ_C may not consists of both a_c and $\bar{a}_c$.

The second requirement $P(\omega, \delta_C) > 0$ in (*ii*) can be deduced to

$$P(\omega, \delta_C) = P(\omega, a_1, a_2, \ldots, a_{|C|}, \bar{a}_{|C|+1}, \ldots, \bar{a}_{|COMPS|}) > 0 \ . \tag{7}$$

Taking together equations (6) and (7), the following logical consequence can be deduced:

$$P(\delta_C) > 0 \ \wedge \ P(\omega, \delta_C) > 0 \Longleftrightarrow SD \cup OBS \cup \Delta_C \nvDash \bot \ . \tag{8}$$

Next, based on Equation (8), the relation between the probabilistic terms of the reduced computational form of Theorem 1 and logical consistency-based reasoning can be established as follows. Since the observations are equal to the set of observed inputs and outputs, these outputs also in the logical framework can be separated into two parts, where the related components is assumed to function abnormally or normally, such as in Bayesian diagnostic problems. Doing so, we obtain the following:

$$SD \cup OBS \cup \Delta \quad \equiv \quad SD \cup O_a \cup O_{\bar{a}} \cup I \cup \Delta \ . \tag{9}$$

Then, according to Equation (9), the correspondence between the probabilistic terms,given in Theorem 1 and logical consistency-based reasoning is the following.

$$\{o_1, o_2, \ldots, \bar{o}_i\}_a \equiv \prod_{q \in Q} p_q^{n_q}(1 - p_q)^{m_q} \quad (10)$$

$$\{o_{i+1}, o_{i+2}, \ldots, \bar{o}_{|COMPS|}\}_{\bar{a}} \equiv b(O_u^a)\ , \quad (11)$$

and term $P(I_u)$ is related to the abnomral and normal functionality of the components, similar to the prior probabilities $P(a_i)$ and $P(\bar{a}_i)$ mentioned above.

Finally, we establish the relations between the deduced computational form of the Bayesian diagnostic reasoning in Theorem 1 and the relations in Equation (10). Theorem 1 exploits independence relations in the representation of Bayesian diagnostic problems, and, therefore, it allows us for probabilistic reasoning with subset $b(O_u^{a''}) \subseteq b(O_u^a)$, which is *not* the case for logical diagnostic reasoning. Therefore, we obtain a computationally simplifi ed expression to obtain consistency-based and, therefore, abductive diagnosis.

5 Conclusion and future work

In this paper, we focused on diagnostic reasoning in Bayesian diagnostic problems using both consistency-based and abductive reasoning. Abductive reasoning poses stronger requirements on diagnostic solutions than consistency-based reasoning. In this paper, we have established a relationship between consistency-based and abductive reasoning with Bayesian diagnostic problems and we showed that abductive diagnoses can only be determined using consistency-based computations. We also established some probabilistic properties regarding model-based Bayesian diagnostic problems, that gave rise to simplifi cations of the computation of the set of consistency-based and abductive diagnoses.

In the future, we would like to explore this theoretical results within an experimental setting.

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